Problem Set V: production

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Exercises will be solved in class on *March 1st, 2010*

1. MWG 5.B.2: homogeneity

Let $f(\cdot)$ be the production function associated with a single-output technology, and let Y be the production set. Show that Y satisfies constant returns to scale if and only if $f(\cdot)$ is homogeneous of degree one.

2. MWG 5.B.3: convexity and concavity

Show that for a single-output technology, Y is convex if and only if the production function $f(z)$ is concave.

3. MWG 5.C.9: profit and supply functions

Derive the profit function $\Pi(p)$ and the supply function (or correspondence) $y(p)$for the following three single-output technologies, whose production functions $f(z)$ are:

- $f(z) = \sqrt{z_1 + z_2}$
- $f(z) = \sqrt{\min\{z_1 + z_2\}}$
- $f(z) = (z_1^\rho + z_2^\rho)^{\frac{1}{\rho}}$

4. Cobb-Douglas production function: all you ever wanted to know

Consider a Cobb-Douglas prduction function, $f(z) = z_1^\alpha z_2^\beta$ with $\alpha, \beta > 0$. For the three cases in which $\alpha + \beta <, =, > 1$:

- draw Y (in 3d), marginal and average product, and the rate of technical substitution (in 2d);
- solve the profit maximisation and the cost minimisation prblems;
- find conditional factor demand functions;
- find supply functions (correspndences);
- find cost functions.