Problem Set IV: UMP, EMP, indirect utility, expenditure

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Exercises will be solved in class on *Monday, February 22nd, 2010*

1. Varian 7.4: UMP-EMP

Consider the indirect utility function given by

$$v(p_1, p_2, w) = \frac{w}{p_1 + p_2}$$

1. What are the Walrasian demand functions?
2. What is the expenditure function?
3. What is the direct utility function?

2. MWG 3.D.6: Stone linear expenditure system

Consider the following utility function in a three-good setting:

$$u(x) = (x_1 - b_1)^{\alpha}(x_2 - b_2)^{\beta}(x_3 - b_3)^{\gamma}$$

Assume that $\alpha + \beta + \gamma = 1$.

1. Write down the FOC for the UMP and derive the consumer's Walrasian demand and the indirect utility function.
2. Verify that these demand functions satisfy the following properties:
   (a) Walrasian demand $x(p, w)$ is homogeneous of degree zero and satisfies Walras' law;
   (b) Indirect utility $v(p, w)$ is homogeneous of degree zero;
   (c) $v(p, w)$ is strictly increasing in w and nonincreasing in p_l for all l;
   (d) $v(p, w)$ is continuous in p and w.

3. MWG 3.G.15: dual properties

Consider the utility function

$$u = 2x_1^{\frac{1}{2}} + 4x_2^{\frac{1}{2}}$$

1. Find the demand functions $x_1(p, w)$ and $x_2(p, w)$
2. Find the compensated demand function $h(p, u)$
3. Find the expenditure function $e(p, u)$ and verify that $h(p, u) = \nabla_p e(p, u)$
4. Find the indirect utility function $v(p, w)$ and verify Roy's identity.