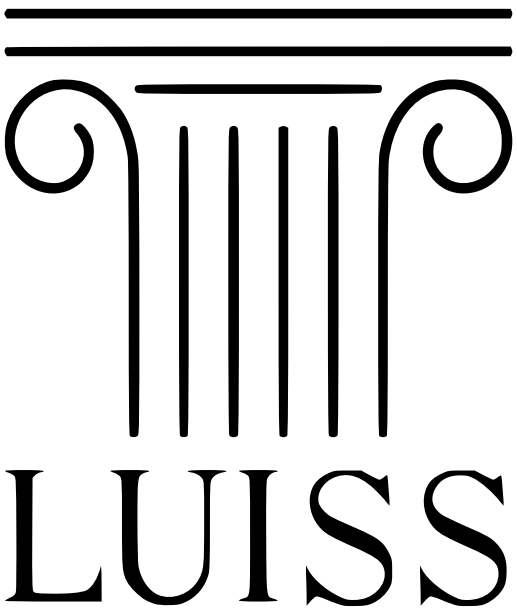
Games and Strategy TA 4

Repeated games



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Recap: repeated games

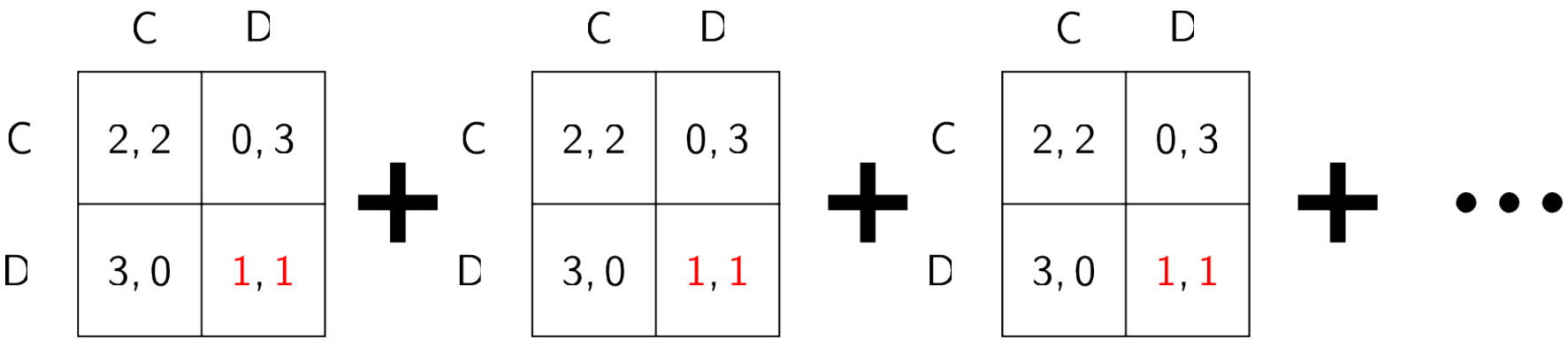


Figure: A repeated standard Prisoner Dilemma

- Is it possible to escape from mutual defection if the game is repeated?
- As in everyday life; does chance of playing again increases likelihood of cooperation?
- Answer: only if repetition infinite (or indefinite); only if players are patient enough.

Recap: discount factor

Discounting assumptions

We assume that the decision maker takes into account the discounted sum of payoffs accruing to her in any period of the supergame Γ made of the infinite repetition of the base game G, if the action profile a is chosen:

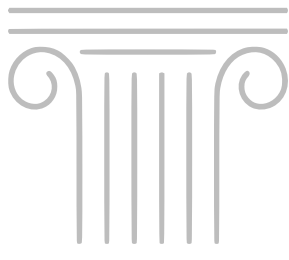
$$U_i(\Gamma) = u_i(a_t) + \delta_i u_i(a_{t+1}) + \delta_i^2 u_i(a_{t+2}) + \cdots + \delta_i^{T-1} u_i(a_T) = \sum_{t=1}^{T} \delta_i^{t-1} u_i(a^t)$$

Why a discount factor?

1. Intrinsic impatience ('meglio un uovo oggi...')
2. Probability of death in future periods
3. Existence of exogenous interest rate
4. (if income grows): decreasing marginal utility of income

One δ to tie them all...

We will assume throughout that the discount factor $\delta_i = \delta$, $\forall i$.



Recap: Important series

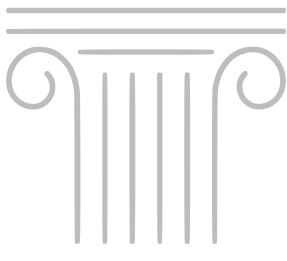
A convergent discount series

$$\sum_{i=k}^{n} \delta^i = \frac{\delta^k - \delta^{n+1}}{1-\delta}$$

This implies some other useful results for infinite series:

$$\sum_{i=0}^{\infty} \delta^i = \frac{1}{1-\delta} \qquad \sum_{i=1}^{\infty} \delta^i = \frac{\delta}{1-\delta}$$

$$\sum_{i=0}^{k} \delta^i = \frac{1-\delta^{k+1}}{1-\delta} \qquad \sum_{i=1}^{k} \delta^i = \frac{\delta-\delta^{k+1}}{1-\delta}$$



Representing strategies

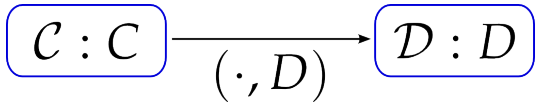


Figure: *grim trigger*

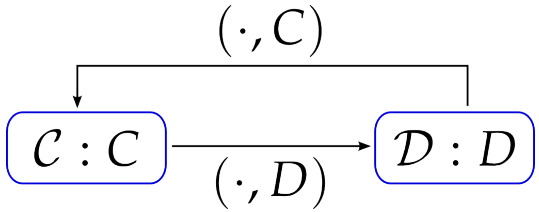


Figure: *tit for tat*

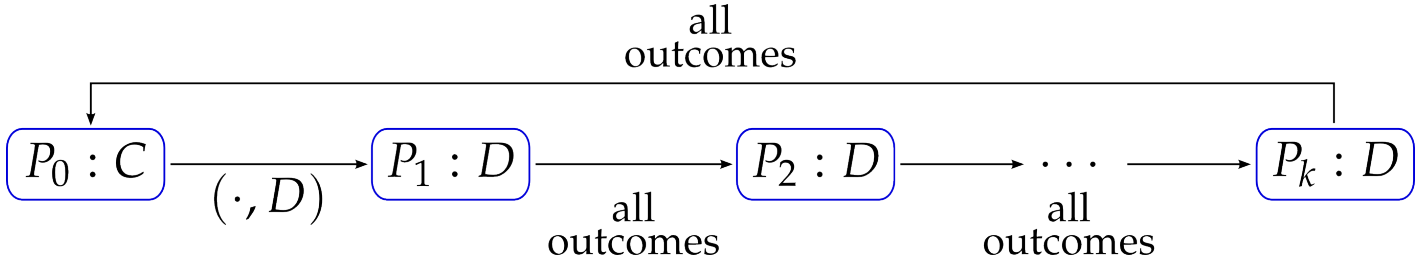
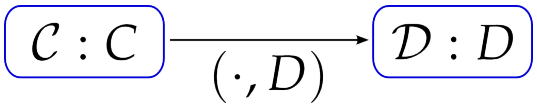


Figure: *k-punishment*

Looking for Nash: *grim trigger*



Looking for Nash

If both use gt, the payoff for both is given by

$$\pi_1(gt) = \pi_2(gt) = 2\frac{1}{1-\delta}$$

Checking for deviations implies:

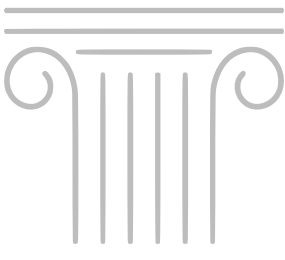
- when is it best to deviate? [first period]
- what is the expected payoff for the deviator? [function of δ]

If player 1 deviates, her payoff is

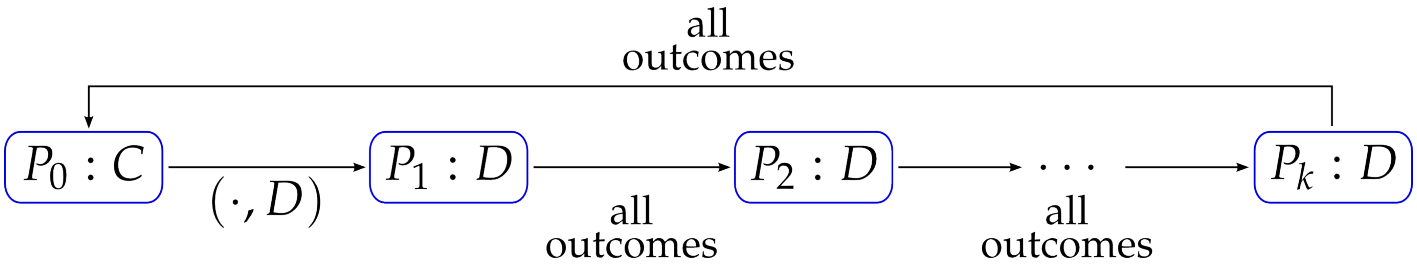
$$\pi_1(D) = 3 + 1\delta + 1\delta^2 + \cdots = 3 + \frac{\delta}{1-\delta}$$

This turns out to be higher than $\pi_1(gt)$ iff

$$\delta < \frac{1}{2} \;\Rightarrow\; \text{Nash iff } \delta \geq \frac{1}{2}$$



Looking for Nash: *k-punishment*



Looking for Nash

We limit attention to the first k periods - after this period, the situation is the same as at start. If both use kp, the payoff for both for the first k periods is given by

$$\pi_1(kp) = \pi_2(kp) = 2\frac{1-\delta^{k+1}}{1-\delta}$$

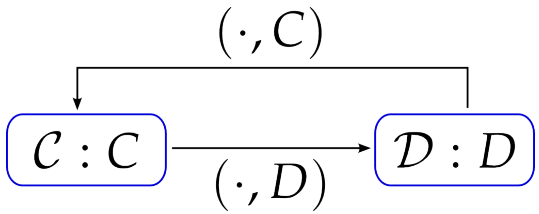
Checking for deviations:
If player 1 deviates, her payoff for the k periods is

$$\pi_1(D) = 3 + \delta + \delta^2 + \cdots + \delta^k = 3 + \frac{\delta - \delta^{k+1}}{1-\delta}$$

This turns out to be lower than $\pi_1(kp)$ iff

$$\delta^{k+1} - 2\delta + 1 \leq 0$$

Looking for Nash: *tit for tat*



Looking for Nash

If both use *tft*, the payoff for both is given by

$$\pi_1(kp) = \pi_2(kp) = 2\frac{1}{1-\delta}$$

Checking for deviations:
If player 1 deviates, she can do so either by alternating, and it that case she gets:

$$\pi_1(D) = 3 + \delta 0 + 3\delta^2 + \cdots = 3 + 3\delta^2 + 3\delta^4 = 3\frac{1}{1-\delta^2}$$

or by deviating once and then sticking to deviation, in which case she gets:

$$\pi_1(D) = 3 + \delta 1 + \delta^2 + \cdots = 3 + \frac{1}{1-\delta}$$

This turns out to in both cases to be lower than $\pi_1(tft)$ iff

$$\delta \geq \frac{1}{2}$$